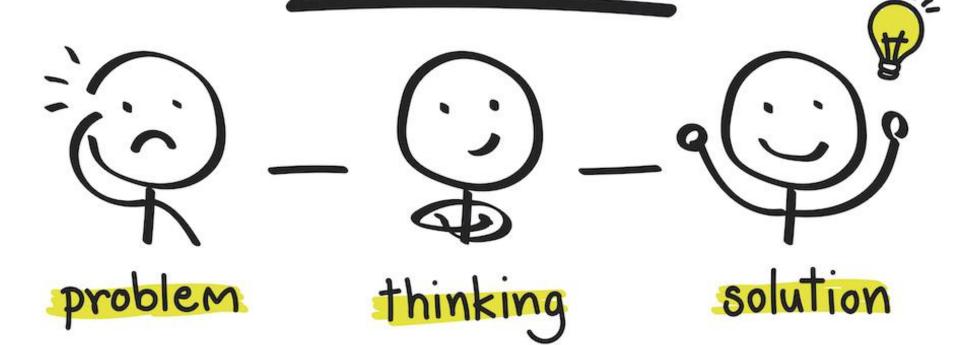


Prepared & Presented by: Mr. Mohamad Seif

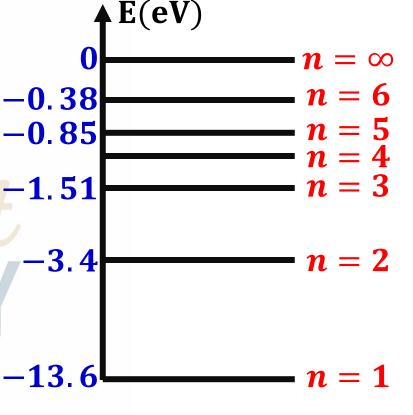
SOLVING



- $1 \ eV = 1.6 \times 10^{-19} J$; $h = 6.62 \times 10^{-34} J.s$; $c = 3 \times 10^8 m/s$
- Figure 1 represents a part of the energy diagram for the energy

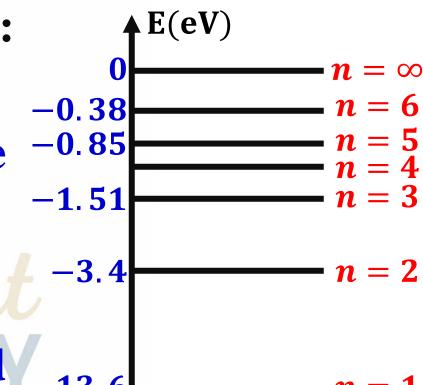
levels of the hydrogen atom.

- 1. Give the value of the energy of the atom:
 - a) Taken in the fundamental state.
 - b) Taken in the ionized state.
- 2) Define the ionization energy of an atom & determine its value for the hydrogen atom.



$$1 \ eV = 1.6 \times 10^{-19} J$$
; $h = 6.62 \times 10^{-34} J.s$; $c = 3 \times 10^8 m/s$

- 1. Give the value of the energy of the atom: a) Taken in the fundamental state.
- The energy of the atom at the fundamental state is $E_1 = -13.6eV$
- b) Taken in the ionized state.

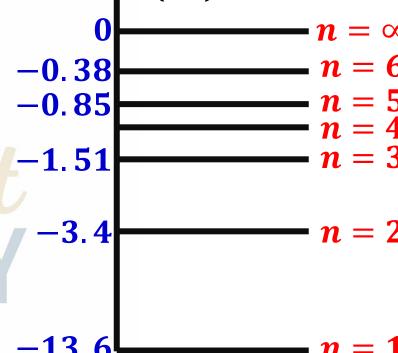


$$1 \text{ eV} = 1.6 \times 10^{-19} \text{J}; h = 6.62 \times 10^{-34} \text{J.s}; c = 3 \times 10^{8} \text{m/s}$$

2) Define the ionization energy of an atom & determine its value for the hydrogen atom.

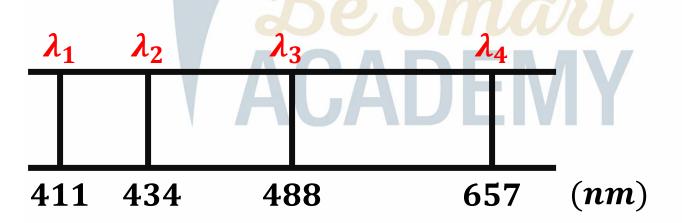
**E(eV)

The ionized energy W_{ion} is the minimum $_{-0.38}$ energy that should be given to the atom $^{-0.85}$ to extract an electron from it.



3) Hydrogen atom has different series of radiations that result from the transitions between its energy levels.

A part of the hydrogen's atom emission spectrum is schematized in figure 2, formed of its visible radiations and corresponds to the downward transition from an energy level ranked n>2 towards the energy level n=2.



a) Justify that the energy, in eV, of a radiation of wavelength λ is given by $E = \frac{1243}{\lambda}$ (λ in nm).

$$E = \frac{hc}{\lambda}$$

$$E = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{\lambda}$$

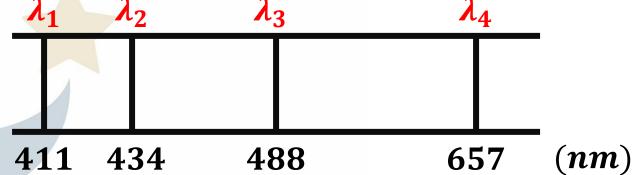
$$E = \frac{1.989 \times 10^{-25}}{\lambda}$$

$$E = \frac{1.989 \times 10^{-25}}{(1.6 \times 10^{-19})(\lambda \times 10^{-9})}$$

$$E=\frac{1243}{\lambda}$$

b) Determine in eV, the energy of each of the 4 radiations emitted. λ_1 λ_2 λ_3 λ_4

$$E_{\lambda_1} = \frac{1243}{411} = 3.02eV$$



$$E_{\lambda_2} = \frac{1243}{434} = 2.86eV_{8}$$

$$CADE^{\lambda_4} = 1.89eV_{1243}$$

$$ACADE^{\lambda_4} = 1.89eV_{1243}$$

$$E_{\lambda_3} = \frac{1243}{488} = 2.55eV$$

c) Deduce the corresponding transitions associated to the emission of each of these radiations.

The transitions corresponds to the downwards transition from an energy level n>2 towards the energy level n=2, E(eV)



4) In what follows, the hydrogen atom is taken in the ground state. Determine, in eV, the energies of the photons absorbed through the transition from the ground state n=1 towards the excited states n=3 & n=4

The energy of photons absorbed to ensure the transition from the ground state n=1 and n=0. The energy of photons absorbed to ensure the transition from the ground state n=1 and n=6. The energy of photons absorbed to ensure n=0. The energy of n=0 and n=0 and n=0. The energy of n=0 and n=0 and

 $E_{vh}=12.09eV$

The energy of photons absorbed to ensure the transition from the ground state n=1 towards the excited states n=4 is:

$$E_{ph} = E_4 - E_1$$

$$E_{ph} = -0.85 - (-13.6)$$

$$E_{ph} = 12.75eV$$

$$E_{ph} = 13.6$$

$$E_{ph} = E_4 - E_1$$

$$E_{ph} = 0.38$$

$$E_{ph} = 0.85$$

$$E_{ph} = 12.75eV$$

5) Deduce whether the hydrogen atom can absorb a photon of energy 12.3eV.

$$E_{ph} = 12.3eV \neq 12.09eV$$

 $E_{ph} = 12.3eV \neq 12.75eV$

Since the energy levels are quantized, then the photon cannot be absorbed.

ACADEMY

6) An incident photon extracts an electron carrying a kinetic energy of 2.1eV. Determine the wavelength of the radiation associated to this photon.

$$E_{ph} = W_{ion} + K.E_{e}$$

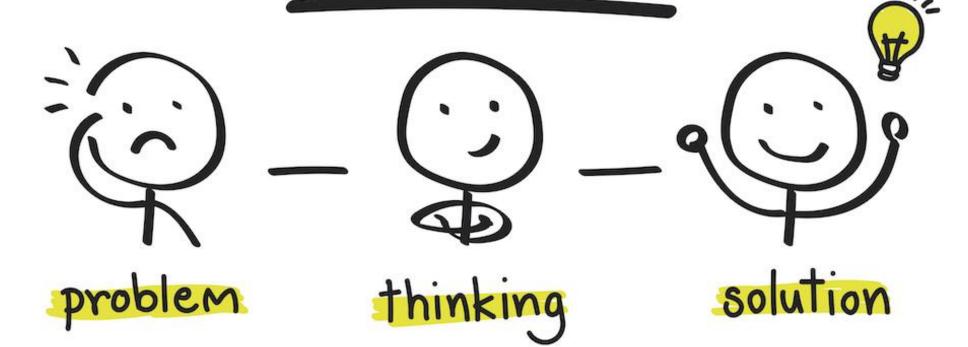
$$\frac{hc}{\lambda} = 13.6 + 2.1$$

$$\frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{\lambda} = 15.7eV \times 10^{-19}$$

$$\lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{15.7eV \times 10^{-19}} = 7.92 \times 10^{-8}m$$



SOLVING



- $c = 3 \times 10^8 m/s$; $1 \text{ eV} = 1.6 \times 10^{-19} J$; $h = 6.62 \times 10^{-34} J.s$
- The energies of the different energy levels of the hydrogen atom are given by the relation $E_n = -\frac{13.6}{n^2}$ (eV).
- A. Energy of the hydrogen atom:
- 1) The energies of the atom are quantized. Justify this using the expression of E_n .
- 2) Determine the energy of the hydrogen atom when it is in the fundamental state and in the second excited state.
- 3) Give the name of the state for which the energy of the atom is zero.

- $c = 3 \times 10^8 m/s$; $1 \text{ eV} = 1.6 \times 10^{-19} J$; $h = 6.62 \times 10^{-34} J.s$
- 1) The energies of the atom are quantized. Justify this using the expression of $E_{\rm n}$.
- The energy of the atom is quantized means that the energy can have specific values only.
- The expression of $E_n = -\frac{13.6}{n^2}$ shows that for every value of n, which is a positive whole number, E has a different value.

$$c = 3 \times 10^8 m/s$$
; $1 \text{ eV} = 1.6 \times 10^{-19} J$; $h = 6.62 \times 10^{-34} J.s$

2) Determine the energy of the hydrogen atom when it is in the fundamental state and in the second excited state.

For fundamental state:
$$n=1$$
 $E_1 = -\frac{13.6}{n^2} = -\frac{13.6}{(1)^2} = -13.6$ eV

For 2nd excited state: n=3
$$E_3 = -\frac{13.6}{n^2} = -\frac{13.6}{(3)^2}$$

$$E_3 = -1.51eV$$

$$c = 3 \times 10^8 m/s$$
; $1 \text{ eV} = 1.6 \times 10^{-19} J$; $h = 6.62 \times 10^{-34} J.s$

3) Give the name of the state for which the energy of the atom is zero.

The energy of the atom is zero when n is infinity, and it is called the ionization state.

For ionization state:
$$n = \infty$$

$$E = \frac{13.6}{n^2} = -\frac{13.6}{(\infty)^2} = 0eV$$

- B. Spectrum of the hydrogen atom:
- Emission spectrum: The Balmer's series of the hydrogen atom is set of radiations of the downward transitions to the level of n = 2. The values of the wavelengths in vacuum of the visible radiations of this series are: 411 nm; 435 nm; 487 nm; 658 nm.
- a) Specify, with justification, the wavelength λ_1 of the visible radiation carrying the greatest energy.
- b) Determine the initial level of the transition giving the radiation of wavelength λ_1 .
- c) Deduce the three initial levels corresponding to the emission of the other visible radiations.

downward transitions to n = 2; 411 nm; 435 nm; 487 nm; 658 nm; $c = 3 \times 10^8 m/s$; $1 \text{ eV} = 1.6 \times 10^{-19} J$; $h = 6.62 \times 10^{-34} J.s$

a) Specify, with justification, the wavelength λ_1 of the visible radiation carrying the greatest energy.

The energy of a photon is given by:

$$E_{ph} = \frac{hc}{\lambda}$$

Then the greatest energy corresponds to the smallest wavelength

downward transitions to n = 2; 411 nm; 435 nm; 487 nm; 658 nm; $c = 3 \times 10^8 m/s$; $1 \text{ eV} = 1.6 \times 10^{-19} J$; $h = 6.62 \times 10^{-34} J.s$

b) Determine the initial level of the transition giving the radiation of wavelength λ_1 .

$$\lambda_1 = 411nm$$

$$E_{\lambda_1} = \frac{nc}{\lambda_1}$$

$$E_{\lambda_1} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{411 \times 10^{-9}}$$

$$E_1 = 4.832 \times 10^{-19} J$$

$$E_1 = \frac{4.832 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$E_1 = 3.024eV$$

downward transitions to n = 2; 411 nm; 435 nm; 487 nm; 658 nm; $c = 3 \times 10^8 m/s$; $1 \text{ eV} = 1.6 \times 10^{-19} J$; $h = 6.62 \times 10^{-34} J.s$

$$E_{i} - E_{2} = E_{\lambda_{1}}$$

$$-\frac{13.6}{n_{i}^{2}} - \left[-\frac{13.6}{(2)^{2}} \right] = 3.024$$

$$-\frac{13.6}{n_{i}^{2}} + \frac{13.6}{4} = 3.024$$

$$-\frac{13.6}{n_i^2} + 3.4 = 3.024$$

$$-\frac{13.6}{n_i^2} = -0.376$$

$$n_i^2 = 36$$

 $n_i = 6$

downward transitions to n = 2; 411 nm; 435 nm; 487 nm; 658 nm; $c = 3 \times 10^8 m/s$; $1 \text{ eV} = 1.6 \times 10^{-19} J$; $h = 6.62 \times 10^{-34} J.s$

c) Deduce the three initial levels corresponding to the emission of the other visible radiations.

$$\lambda_2 = 435nm$$

$$E_{\lambda_2} = \frac{hc}{\lambda_2}$$

 $E_{\lambda_2} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{435 \times 10^{-9}}$

$$\mathbf{E}_{\lambda_2} = \mathbf{0.045} \times \mathbf{10}^{-17} J$$

$$\mathbf{E}_{\lambda_2} = \frac{0.045 \times 10^{-17}}{1.6 \times 10^{-19}}$$

$$E_{\lambda_2} = 2.8125eV$$

$$E_{i} - E_{2} = E_{\lambda_{2}}$$

$$-\frac{13.6}{n_{i}^{2}} - \left[-\frac{13.6}{(2)^{2}} \right] = 2.8125$$

$$-\frac{13.6}{n_{i}^{2}} + \frac{13.6}{4} = 2.8125$$

$$-\frac{13.6}{n_i^2} + 3.4 = 2.8125$$

$$-\frac{13.6}{n_i^2} = -0.5875$$

$$n_i^2 = 25$$

$$n_i = 5$$

For
$$\lambda_3 = 487nm$$

$$E_{\lambda_3} = \frac{hc}{\lambda_2}$$

$$E_{\lambda_3} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{487 \times 10^{-9}}$$

$$\mathbf{E}_{\lambda_3} = 4.078 \times 10^{-19} J$$

$$E_{\lambda_3} = \frac{4.078 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$E_{\lambda_3} = 2.548eV$$

$$\boldsymbol{E_i} - \boldsymbol{E_2} = \boldsymbol{E_{\lambda_3}}$$

$$-\frac{13.6}{n_i^2} - \left[-\frac{13.6}{(2)^2} \right] = 2.548$$

$$-\frac{13.6}{n_i^2} + \frac{13.6}{4} = 2.548$$

$$-\frac{13.6}{n_i^2} + 3.4 = 2.548$$

$$-\frac{13.6}{n_i^2} = -0.852$$

$$n_i = 4$$

For
$$\lambda_4 = 658nm$$

$$E_{\lambda_4} = \frac{hc}{\lambda_4}$$

$$E_{\lambda_4} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{658 \times 10^{-9}}$$

$$\mathbf{E}_{\lambda_4} = 3.018 \times 10^{-19} J$$

$$\mathbf{E}_{\lambda_4} = \frac{3.018 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$E_{\lambda_4} = 1.886eV$$

$$\boldsymbol{E_i} - \boldsymbol{E_2} = \boldsymbol{E_{\lambda_4}}$$

$$-\frac{13.6}{n_i^2} - \left[-\frac{13.6}{(2)^2} \right] = 1.886$$

$$-\frac{13.6}{n_i^2} + \frac{13.6}{4} = 1.886$$

$$-\frac{13.6}{n_i^2} + 3.4 = 1.886$$

$$-\frac{13.6}{n_i^2} = -1.513$$

$$n_i = 3$$

- C. Interaction photon hydrogen atom:
- 1. We send to a hydrogen atom, being in the fundamental state, separately, two photons of respective energies 3.4 eV and 10.2 eV. Specify, with justification, the photon that is absorbed.
- 2. A hydrogen atom found in its fundamental state absorbs a photon of energy 14.6 eV. The electron is thus ejected.
 - a) Justify the ejection of the electron.
 - b) Calculate, in eV, the Kinetic energy of the ejected electron.

1. We send to the hydrogen atom, being in the fundamental state, separately, two photons of respective energies 3.4 eV and 10.2 eV. Specify, with justification, the photon that is absorbed.

$$E_1 = -13.6eV$$
; $E_2 = -3.4eV$; $E_3 = -1.51eV$; $E_4 = -0.85eV$.

There are no two levels having a difference of 3.4 eV, so the photon whose energy is 3.4 eV can't be absorbed.

 $E_2 - E_1 = -3.4 - (-13.6) = 10.2 eV$: Then the photon whose energy is 10.2 eV can be absorbed.

- 2. A hydrogen atom found in its fundamental state absorbs a photon of energy 14.6 eV. The electron is thus ejected. a) Justify the ejection of the electron.
- To extract an electron, the atom should receive an energy which takes it from n=0 to $n=\infty$

$$W_i = E_{\infty} - E_1 \implies W_i = 0 - (-13.6) \implies W_i = +13.6eV$$

So, the energy of the photon (14.6 eV) is sufficient to extract the electron.

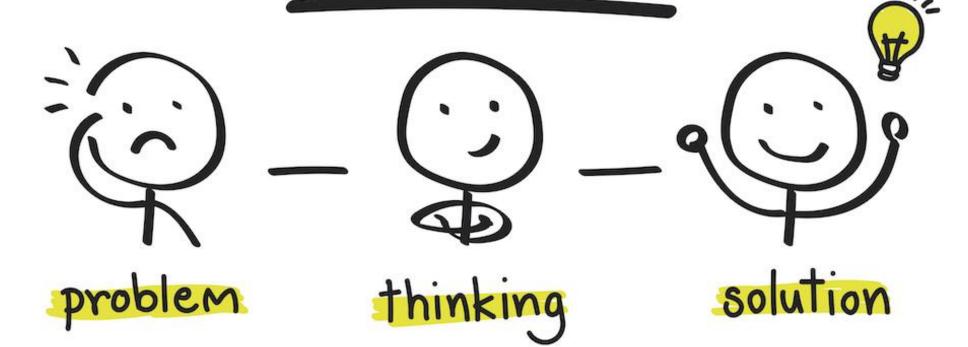
b) Calculate, in eV, the Kinetic energy of the ejected electron. The energy of the ejected electron determined by Einstein relation:

$$E_{ph} = W_i + KE_{ele}$$
 $KE_{ele} = E_{ph} - W_i$
 $KE_{ele} = 14.6 - 13.6$

$$KE_{ele} = 1eV$$



SOLVING

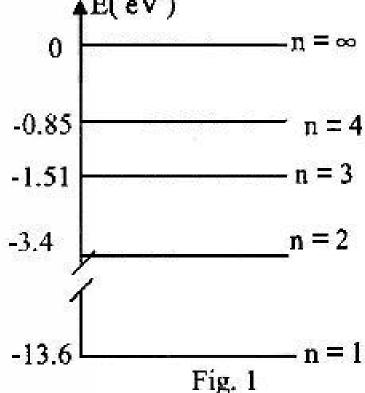


Exercise 3 Energy levels of hydrogen atom

The great Orion Nebula is composed of four very hot stars emitting ultraviolet radiation whose wavelength in vacuum is less than 91.2 nm, within a large <<cloud>> of interstellar gas formed mainly of hydrogen atoms.

The diagram of figure 1 represents some of the energy levels E_n of the hydrogen atom.

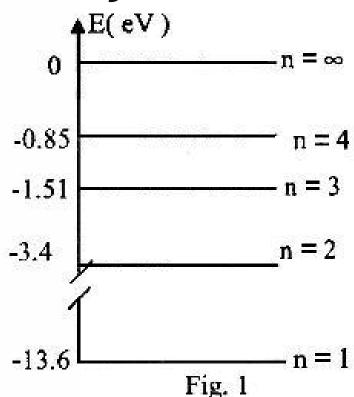
h = $6.62 \times 10^{-34} J.s$; $c = 3 \times 10^8 m/s$; $1 \, eV = 1.6 \times 10^{-19} J$; spectrum of rosy color: $640 \, \text{nm} < \lambda < 680 \, \text{nm}$. visible spectrum : $400 \, \text{nm} < \lambda < 800 \, \text{nm}$



Part A: The hydrogen atom is in its fundamental [ground] state.

1) Show that the minimum value of the energy needed to ionize this atom is equal to $E_i = 2.178 \times 10^{-18} J$.

2) Calculate the wavelength λ_i of the wave associated to the photon whose energy is equal to E_i .



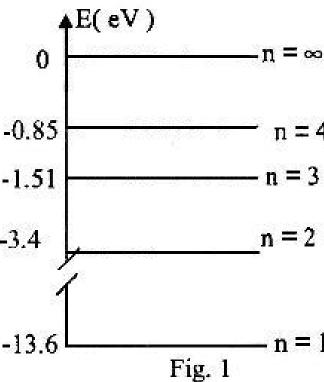
 $h = 6.62 \times 10^{-34} I.s; c = 3 \times 10^8 m/s; 1 eV = 1.6 \times 10^{-19} I; spectrum$ of rosy color: 640 nm $<\lambda$ <680 nm. visible spectrum : 400 nm $<\lambda$ <800 nm

1)Show that the minimum value of the energy needed to ionize this atom is equal to $E_i = 2.178 \times 10^{-18} J$.

$$E_i = E_{\infty} - E_1 \implies E_i = 0 - (-13.6)$$

$$E_i = 13.6 eV \times 1.6 \times 10^{-19}$$

$$E_i = 2.178 \times 10^{-18} J$$



- $h = 6.62 \times 10^{-34} I.s; c = 3 \times 10^8 m/s; 1 eV = 1.6 \times 10^{-19} I; spectrum$ of rosy color: 640 nm $<\lambda$ <680 nm. visible spectrum : 400 nm $<\lambda$ <800 nm
- 2) Calculate the wavelength λ_i of the wave associated to the photon whose energy is equal to E_i .

$$E_{i} = \frac{hc}{\lambda_{i}}$$

$$\lambda_{1} = \frac{hc}{E_{i}}$$

$$\lambda_{i} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{2.178 \times 10^{-18}}$$

$$\lambda_i = 91.24 \times 10^{-9} m$$

Part B: ionized gas in the Orion Nebula

- The interstellar gas in the Orion Nebula being ionized, some extracted electrons are captured by protons at rest (ionized hydrogen atoms) to form hydrogen atoms in an excited state. An excited hydrogen atom undergoes then a progressive downward transition.
- Out of the possible transitions, we consider the transition of the atom from level 3 to level 2.
- 1) Calculate the wavelength, in vacuum, of the radiation corresponding to this transition.
- 2) This radiation is visible. Why?

1) Calculate the wavelength, in vacuum, of the radiation corresponding to this transition.

First method:

$$E_{ph} = E_3 - E_2$$
 \Rightarrow $E_{ph} = -\frac{13.6}{3^2} - (-\frac{13.6}{2^2})$

$$E_{ph} = 1.88eV$$

Second method:

$$\mathbf{E_{ph}} = \mathbf{E_3} - \mathbf{E_2} \implies \mathbf{E_r}$$

$$E_{ph} = E_3 - E_2 \implies E_{ph} = -1.51 - (-3.4)$$

$$E_{ph} = 1.88eV$$

Energy levels of hydrogen atom

$$E_{ph} = \frac{hc}{\lambda}$$

$$\lambda = \frac{6.62 \times 10^{-34} \times 3 \times 10^{9}}{1.88 \times 1.6 \times 10^{-19}}$$

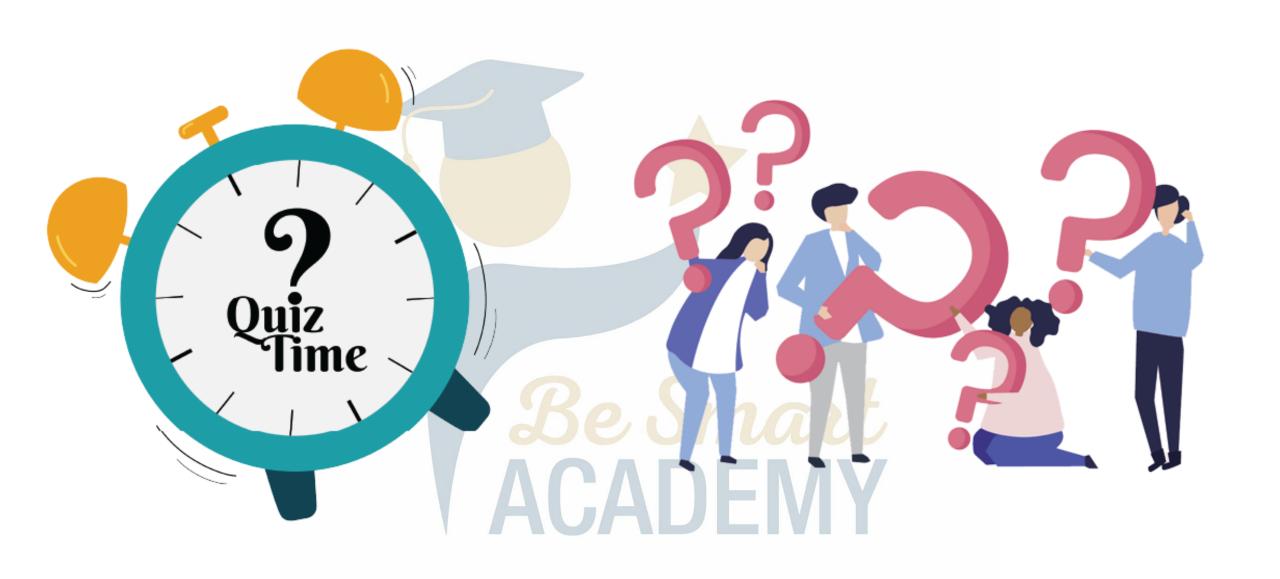
$$\lambda = 657.3 \times 10^{-9} m = 657.3 nm$$

2) This radiation is visible. Why?

Since the its wavelength is in the visible region

 $400nm < \lambda = 657.3nm < 800nm$





The energies of the various levels of the hydrogen atom are given by the relation: $E_n = -\frac{E_0}{n^2}$, where E_0 is a positive constant and n is a positive whole number. Given:

- $h = 6.6 \times 10^{-34} \text{J. s}; 1 \text{eV} = 1.6 \times 10^{-19} \text{J}; C = 3 \times 10^8 \text{m/s}$
- A convenient apparatus (D) is used to detect the electrons.
- 1) The energy of the hydrogen atom is quantized. What is meant by "quantized energy"?
- 2)Explain why the absorption or emission spectrum of hydrogen consists of lines.

- 3) A hydrogen atom, initially excited, undergoes a downward transition from the energy level E_2 to the energy level E_1 . It then emits the radiation of wavelength in vacuum: $\lambda_{2\to 1} = 1.216 \times 10^{-7} m$.
 - a) Determine, in J, the value of the constant E_0 .
 - b)Determine in J the value of the ionization energy of the hydrogen atom taken in its ground state.

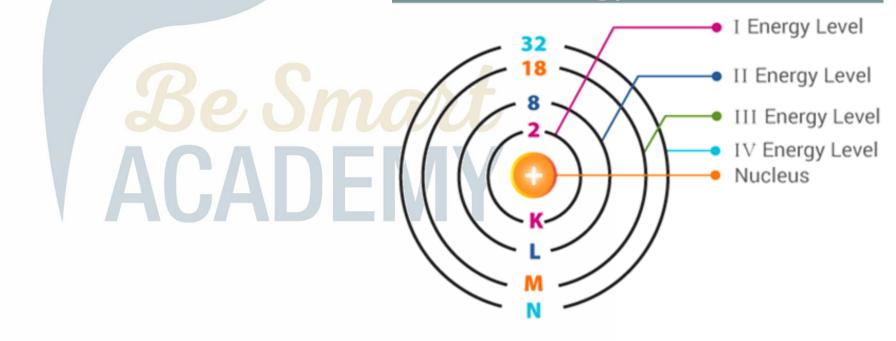
VACADEMY

1) What is meant by "quantized energy"?

According to the relation: $E_n = -\frac{E_0}{n^2}$

The energies of the hydrogen atom can take only well-defined values (discrete), so it is quantized.

Energy Level



2) Explain why the absorption or emission spectrum of hydrogen consists of lines.

For an electronic transition, the emitted photon (or absorbed) has a wavelength Absorption Lines

$$\lambda = \frac{hc}{E_m - E_n}$$

Emission Lines

Since the energies are quantized, this means that the λ has a well determined value, which corresponds to a line.

$$h = 6.6 \times 10^{-34} \text{J. s}; 1 \text{eV} = 1.6 \times 10^{-19} \text{J}; C = 3 \times 10^8 \text{m/s}$$

3)A hydrogen atom, initially excited, undergoes a downward transition from the energy level E_2 to the energy level E_1 . It then emits the radiation of wavelength in vacuum: $\lambda_{2\to 1} = 1.216 \times 10^{-7} m$. a)Determine, in J, the value of the constant E_0 .

$$E_2 = \frac{E_0}{n^2} = -\frac{E_0}{2^2} = -\frac{E_0}{4}$$
 and $E_1 = \frac{E_0}{n^2} = -\frac{E_0}{1^2} = -E_0$

$$E_2 - E_1 = -\frac{E_0}{4} - (-E_0)$$

$$h = 6.6 \times 10^{-34} \text{J. s}; 1 \text{eV} = 1.6 \times 10^{-19} \text{J}; C = 3 \times 10^8 \text{m/s}$$

$$\frac{3E_0}{4} = \frac{hc}{\lambda_{2-1}}$$



$$E_0 = \frac{4hc}{3\lambda_{2-1}}$$

$$E_0 = \frac{4 \times 6.6 \times 10^{-34} \times 3 \times 10^8}{3 \times 1.216 \times 10^{-7}}$$

$$E_0 = 2.17 \times 10^{-18} J$$

$$h = 6.6 \times 10^{-34} \text{J. s}; 1 \text{eV} = 1.6 \times 10^{-19} \text{J}; C = 3 \times 10^{8} \text{m/s}$$

b)Determine in J the value of the ionization energy of the hydrogen atom taken in its ground state

$$E_{ion} = E_{\infty} - E_{1}$$



$$E_{ion} = 0 - (-E_0)$$

$$E_{ion} = E_0 = 2.17 \times 10^{-18} J$$

- 4) Among the series of hydrogen is Balmer, which is characterized by the downward transitions from the energy level $E_P > E_2$ (p > 2) to the energy level E_2 (n = 2). To each transition $P \rightarrow 2$ corresponds a line of wave $\lambda_{p \rightarrow 2}$.
 - a) Show that $\lambda_{p\to 2}$, expressed in nm is given by $\frac{1}{\lambda_{n\to 2}}$ =

$$1.096 \times 10^{-2} \left[\frac{1}{4} - \frac{1}{p^2} \right]$$
. See Smart

b)Show that the wavelengths of the corresponding radiations tend, when $p \to \infty$, towards a limit λ_0 whose value is to be calculated.

a) Show that $\lambda_{p\to 2}$, expressed in nm is given by $\frac{1}{\lambda_{p\to 2}}=1.096$

$$\times 10^{-2} \left[\frac{1}{4} - \frac{1}{p^2} \right].$$

$$E_{ph} = E_P - E_2$$

$$\frac{hc}{\lambda_{p\to 2}} = -\frac{E_0}{p^2} + \frac{E_0}{2^2}$$

$$\frac{hc}{hc} = E_0 \left[\frac{1}{4} - \frac{1}{p^2} \right]$$

$$\frac{1}{\lambda_{p\to 2}} = \frac{E_0}{hc} \left[\frac{1}{4} - \frac{1}{p^2} \right]$$

$$\frac{1}{\lambda_{p\to 2}} = \frac{2.17 \times 10^{-18}}{6.62 \times 10^{-34} \times 3 \times 10^{8}} \left[\frac{1}{4} - \frac{1}{p^2} \right]$$

$$\frac{1}{\lambda_{p\to 2}} = 1.096 \times 10^{-2} \left[\frac{1}{4} - \frac{1}{p^2} \right]$$

b)Show that the wavelengths of the corresponding radiations tend, when $p \to \infty$, towards a limit λ_0 whose value is to be calculated

$$\frac{1}{\lambda_{p\to 2}} = 1.096 \times 10^{-2} \left[\frac{1}{4} - \frac{1}{p^2} \right]$$

$$\frac{1}{\lambda_0} = 1.096 \times 10^{-2} \left[\frac{1}{4} - \frac{1}{\infty^2} \right] \longrightarrow \frac{1}{\lambda_0} = 1.096 \times 10^{-2} \left[\frac{1}{4} - 0 \right]$$

$$\frac{1}{\lambda_0} = 0.274 \times 10^{-2}$$



$$\lambda_0 = 364.9nm$$

